MICROWAVE CIRCUIT ANALYSIS BY SPARSE MATRIX TECHNIQUES

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Abstract

Linear microwave circuit analysis by sparse matrix techniques is discussed. Optimum equation ordering and pivoting are proposed to reduce execution time, storage requirement and to improve accuracy. Details on the implemented program and a numerical example are given.

Introduction

A computer program for analyzing linear microwave circuits can draw great advantage from sparse matrix techniques which, associated with a code generation program, allow computing and storing of only the nonzero elements. In this paper such an approach is investigated in order to reduce execution time and storage requirements, as well as to choose optimum pivots.

The computer program which has been written to calculate, in terms of component scattering parameters, incident and reflected waves at any component port, is also described, and a numerical example is given.

Problem Formulation

For every microwave circuit component with n_k ports a system of n_k equations can be written:

$$\mathbf{b}_k = \mathbf{S}_k \, \mathbf{a}_k \quad , \tag{1}$$

$\mathbf{S}_k$ being its scattering matrix, $\mathbf{a}_k$ and $\mathbf{b}_k$ the vectors of incident and reflected waves at its n_k ports. A generator, instead, is described by relation:

$$b_g = S_g \, a_g + c_g \quad , \tag{2}$$

to take the impressed wave c_g into account, too.

Collecting together the equations relative to all the m components and generators, a system describing the circuit with all the elements uncoupled is obtained:

$$\mathbf{b} = \mathbf{S}\mathbf{a} + \mathbf{c} \quad , \tag{3}$$

where:

$$\mathbf{a} = \begin{vmatrix} \mathbf{a}_1 \\ \cdot \\ \mathbf{a}_i \\ \cdot \\ \mathbf{a}_m \end{vmatrix} , \quad \mathbf{b} = \begin{vmatrix} \mathbf{b}_1 \\ \cdot \\ \mathbf{b}_i \\ \cdot \\ \mathbf{b}_m \end{vmatrix} , \quad \mathbf{c} = \begin{vmatrix} \mathbf{c}_1 \\ \cdot \\ \mathbf{c}_i \\ \cdot \\ \mathbf{c}_m \end{vmatrix} , \tag{4}$$

$$\mathbf{S} = \begin{vmatrix} \mathbf{S}_1 & \cdot & \mathbf{0} & \cdot & \mathbf{0} \\ \cdot & & \cdot & & \cdot \\ \mathbf{0} & \cdot & \mathbf{S}_i & \cdot & \mathbf{0} \\ \cdot & & \cdot & & \cdot \\ \mathbf{0} & \cdot & \mathbf{0} & \cdot & \mathbf{S}_m \end{vmatrix} ,$$

$\mathbf{a}_i$, $\mathbf{b}_i$, $\mathbf{c}_i$ being incident, reflected and impressed wave vectors relative to the i-th component and $\mathbf{S}_i$ its scattering matrix.

The connections between various components imposed by circuit topology introduce constraints to incident and reflected waves at adjacent ports which may be put in the form:

$$\mathbf{b} = \mathbf{\Gamma}\mathbf{a} \quad , \tag{5}$$

$\mathbf{\Gamma}$ being the connection matrix[1, 2]: its elements are all zeros except those in the entries corresponding to adjacent ports, which are 1's if normalization numbers are the same.

From (3) and (5), by setting:

$$\mathbf{M} = \mathbf{\Gamma} - \mathbf{S} \quad , \tag{6}$$

one obtains:

$$\mathbf{a} = \mathbf{M}^{-1}\mathbf{c} , \quad \mathbf{b} = \mathbf{\Gamma}\mathbf{M}^{-1}\mathbf{c} \quad , \tag{7}$$

which completely describe circuit behavior and allow determination of the waves $\mathbf{a}$ and $\mathbf{b}$ at all the component ports when impressed waves $\mathbf{c}$ are given.

When the number of component ports is large the solution of system (7) requires a lot of computations if sparsity of $\mathbf{M}$ and $\mathbf{c}$ is not taken into account. In fact, $\mathbf{M}$ is a matrix which has most of its elements zero; sparse matrix solution techniques can be usefully applied especially when the system must be solved many times, with fixed sparseness structure but with different numerical values for the $\mathbf{M}$ elements, as happens in frequency domain analysis and in optimization process.

Sparse Matrix Solution and Equation Ordering

A discussion on the properties of matrix $\mathbf{M}$ is given in this section in order to find the most convenient technique to solve system (7).

The computational effort can be greatly reduced by taking into account the following:

- the only nonzero elements of matrix $\mathbf{M}$ are: the diagonal ones, which are the reflection coefficients at component ports; those in the entries corresponding to ports belonging to the same component, which are the transmission coefficients between the two ports; and those corresponding to ports connected together, which take the constant value 1;
- sparseness structure of $\mathbf{M}$ is fixed and does not depend on the frequency;
- numerical values of nonzero elements may change with the frequency except the 1's indicating connections;
- vector $\mathbf{c}$ has nonzero terms only in positions rela-

tive to generators.

The fixed sparseness structure of **M** and **c** makes the Crout method very convenient for solving system (7) especially when it has to be solved many times with changed coefficient values. The method consists of factorizing **M** into two matrices:

$$\mathbf{M} = \mathbf{L}\mathbf{U} \ , \qquad (8)$$

L lower triangular and **U** upper triangular with 1's on the diagonal. Then, by the forward and back substitutions:

$$\mathbf{L}\mathbf{y} = \mathbf{c} \ , \qquad \mathbf{U}\mathbf{a} = \mathbf{y} \ , \qquad (9)$$

derived from (7), all the wave variables may be obtained.

The elements of matrices **L** and **U** are determined by the following recurrent formulae[3]:

$$l_{ik} = m_{ik} - \sum_{\mu=1}^{k-1} l_{i\mu} u_{\mu k} \ , \qquad i \geq k \qquad (10)$$

$$u_{kk} = 1 \ , \qquad (11)$$

$$u_{kj} = \left(m_{kj} - \sum_{\mu=1}^{k-1} l_{k\mu} u_{\mu j} \right) / l_{kk} \ , \qquad j<k \qquad (12)$$

for $k = 1, 2, \ldots, n$ where $n = \Sigma_1^m n_i$ is matrix dimension.

The forward and back substitutions give:

$$y_j = \left(c_j - \sum_{\mu=1}^{j-1} l_{j\mu} y_\mu \right) / l_{jj} \ , \qquad j=1,2,\ldots n \qquad (13)$$

$$a_j = y_j - \sum_{\mu=j+1}^{n} u_{j\mu} a_\mu \ , \qquad j=n, n-1, \ldots 1 \qquad (14)$$

All the elements of **L** and **U** can be stored in a matrix:

$$\mathbf{T} = \mathbf{L} + \mathbf{U} - \mathbf{E} \ ,$$

E being unity matrix. Any t_{jk} of **T** is zero if both m_{jk} of **M** and all the products $t_{ji}\, t_{ik}$ with: $1 \leq i \leq \min\{j-1, k-1\}$, are zero. Therefore, the number of nonzeros in **T** depends on the ordering of rows and columns in **M**, as is discussed later.

A great reduction in execution time is obtained with the reduced Crout method[4] according to which only the nonzero operands are considered in computing the nonzeros of **T**. The strategy followed consists of generating a FORTRAN code which contains the statements strictly necessary to execute the required operations.

Since the code depends only on the matrix **M** structure, it is generated only once before starting the execution of arithmetical operations. In order to minimize the code length, which depends on the sparseness structure of **T**, row and column ordering of **M** has to be performed before generating the code; particular attention must, however, be paid because precision depends on the values of the diagonal elements of the re-ordered matrix, the pivots, which are used as divisors in (12) and (13).

This predetermined pivoting might cause a loss of accuracy, due to round-off errors, for some frequency points, since the values of component parameters change with the frequency. However, in system (7) every row of **M** contains the constant 1, deriving from **Γ**, which could be an ideal pivot because it allows great precision, independent of frequency and, at the same time, divisions are avoided. Really, half the 1's are modified in the course of the factorization process but rarely may their value become zero and only in anomalous cases (e.g. when instability occurs).

Many ordering strategies have been proposed by different authors with reference to the nodal admittance matrix[5,6]. Some of these have been proved, together with others which were set-up specifically for the matrix **M**, by a program which gives the ratio between the nonzeros in **T** and in **M**, which may be assumed as the index of algorithm efficiency.

The following strategy was the most convenient, due to its low index (equal to or less than all the others) and for the simplicity of implementation:

- the couple of rows relative to adjacent ports are considered together and ordered so that each couple has a number of nonzeros not greater than that of the successive one; in every couple the row with fewer nonzeros precedes the other;
- the columns are then ordered to place all the 1's of **Γ** on diagonal.

This algorithm has been adopted in the program for microwave circuit analysis, which has been implemented and is described in the next section.

Program Description

The program has a two-pass compiler-like structure. It translates a circuit topological description into a FORTRAN code containing all the operations whose execution gives incident wave vector **a** in terms of component S-parameters.

On the basis of a circuit topological description, the first pass determines the optimum ordering of equations by the given algorithm and produces an intermediate file containing a description of the nonzero positions in the ordered matrix **M** and a map of component S-parameters. The second pass generates a FORTRAN code formed by factorization and substitution routines and their main program. Then, after compilation of the generated code, an arithmetical phase begins which consists of code execution for every set of parameter values corresponding to different frequency points, or to different steps of an optimization process.

The whole program has been structured so that the memory waste be minimized by storing all the information in pseudo-dynamically allocated tables.

Conclusion

A method for analyzing microwave circuits in terms of scattering parameters by sparse matrix solution has been proposed and the implemented program has been described.

In order to give quantitative information on the program the thin-film strip-line branching-filter in fig. 1a has been analyzed. It has been described for the program as shown in fig. 1b with port 1 connected to a generator, ports 2 and 3 to loads and all the others to open-circuit terminations. In fig. 2 the computed transmission coefficients $|S_{21}|$ and $|S_{31}|$ are plotted vs. frequency.

In the given example, **M** is a 96x96 matrix with 384 nonzeros. The CDC6600 CPU time for ordering and for generating and compiling the factorization and substitution code, which is composed of 1410 statements, is about 10 secs. The execution time for evaluating the normalized waves at all the ports is about 13 msecs for every frequency point.

The thin-film coupled-line component S-parameters have been determined from geometrical description by subroutines[7] associated to the program.

Maximum memory occupation, which is of 30k words, was required during generated code compilation and includes the 16k words of the compiler.

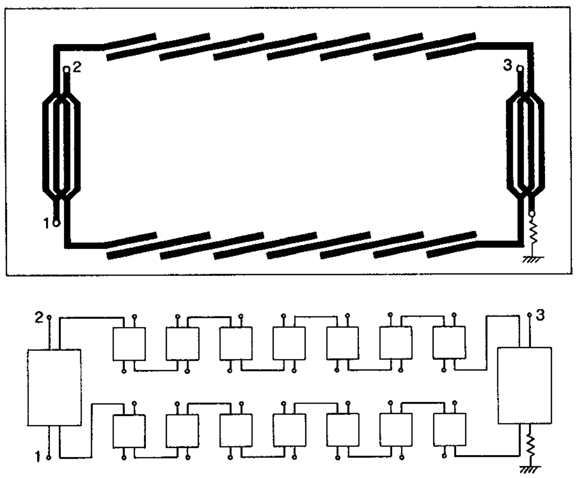

Fig. 1 a) Thin-film strip-line branching-filter;
b) Equivalent circuit as described for the program.

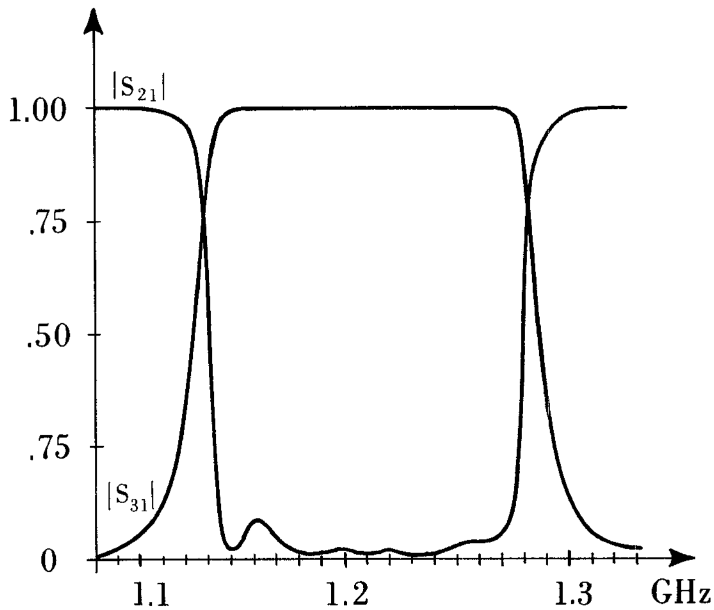

Fig. 2 Transmission coefficients $|S_{21}|$ and $|S_{31}|$, vs. frequency.

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